# A PREFERENTIAL, TOTALLY PROPORTIONAL ELECTORAL SYSTEM WITHOUT GEOGRAPHICAL DIVISIONS 

Stéphane Rouillon, B.Ing., M.Sc.A.,<br>Département de Mathématiques Appliquées, École Polytechnique de Montréal (GÉRAD), stephane.rouillon@sympatico.ca

Abstract. This electoral system offers numerous advantages:

- Each elector needs to go to his or her polling station only once to vote.
- Votes in support of a specific policy cannot be diluted by the fact that several candidates are running to defend that policy.
- An elector can cast a blank ballot to protest against all the proposed candidates. This result is interpreted differently from the result produced by absent voters or individuals who are unfit to vote.
- Computerization makes it possible to maintain the current timeframe for the release of results. Retention of the hardcopy prevents fraud.
- A multitude of candidates can run for office, including independent candidates. Hence many different ideas can be confronted in accordance with democratic principles.
- Allowing voters to rally to the support of a candidate gives an opportunity to regroup to both the party in power and the opposition. Despite the large number of candidates, it is quite likely that the parliament will only include a few parties.
- The "crutch" option ensures a majority government for a reduced term, instead of a shaky coalition.
- Each riding could represent an equal portion of the population, to within one voter.
- The Direction Générale des Élections, or its equivalent outside Québec, does not have to redefine the boundaries of electoral divisions between elections, a process both costly and debatable.
- Each elected official represents a disseminated sample of the population, which encourages him or her to defend common interests. Therefore, ministers have no political interest in favouring any specific group when deciding on the location of a plant, an airport, a hospital or a museum for example. This system subjects the country/province/municipality to the same rules to a greater extent than it promotes exceptions by region.
- It is very unlikely that a candidate would be attributed the same electors as in a previous election. In this way, a party leader or a minister is not always elected by the same voters.
- The model generally attributes one elected official per riding, which means voters continue to feel represented by "their" candidate.
- This model's proportional representation is optimal. The error between votes cast and seat distribution is minimized. Using this seat distribution for the parties, the number of votes collected by elected officials is then maximized.

Note: It works with geographical divisions too, but it is less effective and less fair to candidates.

For concision purposes, the masculine form was used in this document.

An election is a representation exercise. Even so, in almost each election in Canada in the last ten years, the voter participation rate has dropped. Why ? It is doubtful that the $40 \%$ of delinquent electors who represent the norm nowadays are all anarchists. The finger should be pointed instead at the First-Past-The-Post (FPTP) or uninominal with one round system, which contributes to the voters' lack of interest.

On the one hand, an elector is often muzzled. A striking example within the current system is the existence of type " $Z$ " ridings that are already determined beforehand. Many electors do not even go to vote when they see the polls already predicting a victory in their riding. The undecided electoral divisions then become the only real stakes for the politicians. This system infringes on one of the fundamental principles of a representation exercise: the vote of each elector should have equal weight.

On the other hand, the current electoral system creates a geographical link between an elected official and his electorate. Some time ago, this was necessary to gather problems particular to a region at the parliament level. However, it encourages "buying" the votes from a targeted electorate without taking the impact on all the electors into account. At each election, announcements of public investments and promises shower down on undecided ridings to the detriment of the common interest. The dealings (or dare we say corruption) tied to the localization of a development project then come to reward those ridings.

To counter voter disengagement, several political scientists have proposed some interesting alternatives to the current system [1-5]. Without criticizing these systems, which do constitute some improvement over the current voting method, it is possible to build a voting system that better meets all expected criteria. After describing the 5 components that make up the suggested model, its main advantages will be presented.

## 1- The preferential vote or ordinal ballot is used

This allows an elector to vote for several election rounds, in only one visit to the polling station. For example:

Riding No. 4
Candidate A
Candidate B 3
Candidate C 1
Candidate D
Candidate E 2
None

In the example above, the elector contributes to riding No. 4 representation by first separating acceptable candidates ( $\mathrm{B}, \mathrm{C}$ and E ) from undesirable candidates ( A and D). Next, he ranks the acceptable candidates according to his preferences: our elector votes for candidate C and indicates that he would be willing to rally to candidate E if C is not available anymore, and later to candidate B if neither C nor E are available. The
"None" box allows electors who feel all candidates are undesirable to clearly express their opinion. Thus, the vote of an elector who wants to vote "Blank" can be differentiated from the vote of an elector unfit to vote. The elector's action is simple and easy to interpret.

## 2- The vote follows the leadership run-off system with rallying

This is also called Alternative Vote (AV) or Instant Runoff Vote (IRV). At each "Round", the elector's vote is attributed to the first candidate still running from his preference list: the candidate with the least votes is then eliminated. At the next round, his votes are redistributed, until only one candidate remains. Each elector only votes once in the final representation. For the final result, his vote is attributed to the last candidate he agreed to rally to. Example of riding No.4:

## 1st Round

Candidate A $32 \%$
Candidate B 29\%
Candidate C 17\%
Candidate D 14\%
Candidate $\mathrm{E} \quad 2 \%===\Rightarrow$ Candidate E is eliminated.
None $\quad 6 \%====>6 \%$ of "None" votes as final result.
2nd Round
Candidate A $32 \%$
Candidate B 30\%
Candidate C 17\%
Candidate $\mathrm{D} \quad 14 \%===>$ Candidate D is eliminated.
None $\quad 7 \%====>7 \%-6 \%=1 \%$ for Candidate $E$ as final result.

3rd Round
Candidate B 34\%
Candidate A $33 \%$
Candidate $\mathrm{C} \quad 17 \%===>$ Candidate C is eliminated.
None $\quad 16 \%====>16 \%-7 \%=9 \%$ for Candidate $D$ as final result.
4th Round
Candidate A $42 \%$
Candidate B $39 \%===>$ Candidate B is eliminated.
None $\quad 19 \%====>19 \%-16 \%=3 \%$ for Candidate C as final result.

5th Round
Candidate A $54 \%===>54 \%$ for Candidate A as final result.
None $\quad 46 \%====>46 \%-19 \%=27 \%$ for Candidate B as final result.

## Final Result:

Candidate A 54\%
Candidate B 27\%
Candidate C $3 \%$
Candidate D $9 \%$
Candidate E 1\%
None $6 \%$

The elector described in point 1 - votes for candidate C in the three first rounds. In the fourth round, since candidate C (his first choice) is eliminated, and candidate E (his second choice) was already eliminated in a previous round, our elector becomes one of the $5 \%(39 \%-34 \%)$ of voters who rally to candidate B (his third choice). In the last round, our elector refuses to rally to candidate A (an unacceptable candidate) and therefore, as a final result, he votes for candidate B, just like $27 \%$ of the electorate.

A similar extension of Ranked Pairs to produce weights as output from each district is possible. In the case of a tie for last place in any round, each scenario is played out and the average of the results is retained as a final result.

## 3- The proportional representation is optimal

It should be noted that, at this stage, no one has been elected yet. Even if a candidate obtains over $50 \%$ of votes as a final result for his riding, his election is not guaranteed. Indeed, if all candidates of a political party receive over $75 \%$ of the votes except for Mr. X who obtains only a flat $50 \%+1$, Mr. X is a very poor representative of his party's philosophy.

Let us examine an example of final results for all ridings (assuming there are 10 ridings in this case). The last column indicates for each party the average of the votes over all ridings:

| Party Riding | n.1 | n.2 | n. 3 | n. 4 | n.5 | n. 6 | n. 7 | n. 8 | n. 9 | n. 10 | Average |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | :---: |
| Party A | 52 | 6 | 85 | 54 | 6 | 12 | 34 | 39 | 33 | 24 | 34.5 |
| Party B | 13 | 32 | 6 | 27 | 19 | 12 | 17 | 32 | 31 | 0 | 18.9 |
| Party C | 9 | 51 | 0 | 3 | 9 | 20 | 19 | 7 | 1 | 22 | 14.1 |
| Party D | 4 | 0 | 2 | 9 | 11 | 22 | 3 | 10 | 23 | 14 | 9.8 |
| Party E | 1 | 0 | 0 | 1 | 4 | 4 | 3 | 2 | 4 | 6 | 2.5 |
| Independent | 0 | 0 | 0 | 0 | 46 | 21 | 10 | 0 | 0 | 23 | 10 |
| None | 21 | 11 | 7 | 6 | 5 | 9 | 14 | 10 | 8 | 11 | 10.2 |

## A) We start by evaluating the number of seats for each party

We want the 10 seats to be distributed proportionally. We calculate the percentage of mean final votes per party equivalent to an available seat by removing blank votes ("None"): $100 \%-10.2 \%=89.8 \%$ for 10 seats, thus $8.98 \%$ per seat. Nevertheless, independent candidates do not constitute a party and are treated separately. We attribute the seats per party by minimizing the representation error. For each party $i$, we note $x_{i}$ the fractional equivalent in seats of its votes and $s_{i}$ the number of seats that are granted to the party. Thus, the error of representation is equivalent in seats wrongly attributed to:

$$
\begin{equation*}
\Delta=\frac{1}{2} \sum_{i}\left|x_{i}-s_{i}\right| \tag{1}
\end{equation*}
$$

| se: |  | $x_{i}$ | vs | $s_{i}$ |
| :---: | :---: | :---: | :---: | :---: |
| Party A | 34.5\% / 8.98\% | 3.84 | vs | 4 el |
| Party B | 18.9\% / 8.98\% = | 2.1 | vs | 2 elected official |
| Party C | 14.1\% / 8.98\% | 1.57 | vs | 2 elected |
| Party D | 9.8\% / 8.98\% | 1.09 | vs | 1 elected official |
| Ind S. 5 | 4.6\% / 8.98\% | 0.51 | vs | 1 elected official |
| Party E | 2.5\% / 8.98\% | 0.28 | vS | 0 elected offic |
| Ind S. 10 | 2.3\% / 8.98\% | 0.26 | vs | 0 elec |
| Ind S. 6 | 2.1\% / 8.98\% | 0.23 | vS | 0 ele |
| Ind S. 7 | 1\% / 8.98\% | 0.11 | vs | 0 e |

The error of representation $\Delta$ is roughly one seat:

$$
\begin{aligned}
& ((4-3.84)+(2.1-2)+(2-1.57)+(1.09-1)+ \\
& (1-0.51)+(0.28-0)+(0.26-0)+(0.23-0)+(0.11-0)) / 2=1.08 \text { seats. }
\end{aligned}
$$

Finally, proxy representation can produce an exactly proportional representation between all parties that elected at least one candidate.

Singleness of the representation: to find the minimal error, round the number of seats to the nearest integer. However, the total number of seats may not necessarily be respected. Hence, we correct the number of seats one by one in order to reach the expected total number of seats ( 10 in our example). If there are not enough elected officials, we increase by one the number of elected officials from the party with the highest fractional part inferior to 0.5 and we repeat using decreasing fractional parts. If there are too many elected officials, we remove one seat from the party with the lowest fractional part superior to 0.5 and we repeat using increasing fractional parts. Of course, an independent candidate cannot end up with two seats... In the event of equal fractional parts, the elected official(s) is (are) the candidate(s) who obtained the most votes as final result. If the equality persists, the party with the highest representation gets an advantage (3.46 seats vs 2.46 seats become respectively 4 seats vs 2 seats). If the equality is exact, the leader of the party with the most votes, other than the ones concerned, picks a winner(s).

## B) Seats of a party are assigned to candidates with the best final results

Thus, several officials can be elected for a riding or none at all. Presented in decreasing order of votes, from the previous example:

The elected officials from party A are its candidates in ridings n. 3 ( $85 \%$ ), n. 4 (54\%), n. 1 (52\%) and n. 8 (39\%).
The elected officials from party B are its candidates in ridings n. 2 (32\%) and n. 8 (32\%). The elected officials from party C are its candidates in ridings n. 2 (51\%) and n. $10(22 \%)$. The elected official from party D is its candidate in riding n. 9 ( $23 \%$ ). The independent in riding n. 5 ( $46 \%$ ) is elected.

Composition of the Parliament:
Party A: Candidates No.1, No.3, No. 4 and No.8.
Party B: Candidates No. 2 and No.8.
Party C: Candidates No. 2 and No. 10.
Party D: Candidate No.9.
Independent No. 5 .
Ridings n. 2 and n. 8 produce two elected officials each and ridings n. 6 and n. 7 none. In general, results should be distributed in a more regular way, but the example shows how the model can solve the worst distortions. It is less visible with this very irregular example, but the ridings with no representative are often those where the electorate refused to rally or voted blank. In the event of equality between candidates of a same party for the final attribution of seats, the leader of the party picks the winner(s).

## 4- A "crutch" option is used to prevent shaky governments

Instead of the shaky, short-lived coalitions of more than two parties that proportional models sometimes give rise to, the following option is proposed to minority governments. Considering that the product ( $n b$ of representative $x$ length of the mandate) is fixed, the party with the most votes can recuperate the minimum number of losing candidates necessary to obtain half (or just less than half) of the seats, thus reducing the maximal length of its mandate. These recovered officials obtain the same status as the others. The parliament will contain more seats until the next general election. The "crutch" is only available in the event of a general election. Letting the plurality party gather just less than half seats would ensure that a coalition governs.

In the previous example, party A can either form a coalition (let us say for 4 years maximum), or take the "crutch". In the latter case, party A will be able to recover its candidates No. 7 (34\%), No. 9 (33\%) and No. 10 (24\%) to get an absolute majority (of 4+3 $=7$ representatives in a chamber of $10+3=13$ representatives). By a simple rule of three, the maximal length of the mandate thus becomes (4 years x 4 representatives / 7 representatives) 2 years and 104 days. The leader of the plurality party in power announces its decision. The maximal length of the mandate and the number of seats forming the parliament are hence fixed until the next general election.

## 5-Electors' assignation per riding is random and prevents any discrimination

This component is necessary even if its revolutionary aspect condemns it to be applied in a future step. Suppose that the parliament (of the country, province or municipality) is composed of 100 seats. The two last digits of your social insurance number designate the riding for which you vote. For other countries or other government levels, birthdays information (days, months and modulo over years) can be used as well. Because there is no particular link between a candidate and his electorate, there can be no promises or favours made in exchange for votes. An elected official who defends the interests of a category of workers or those of any group does it necessarily because of his convictions and not personal interest. When using a geographical representation, we always accuse - rightly or wrongly - the representative(s) of the region of looking after his (their) own interests. Representatives of under-populated regions, being a minority, always have to deal with the other elected officials who, having nothing to gain, are not interested in their problems. Without geographical divisions, these regions will then be able to find several objective and neutral defendors when an injustice exists.

This step is particulary crucial for a heterogeneous electorate. Thus, the fluctuations of votes between candidates of the same party do not measure the trends between circumscriptions, but rather the different positions and priorities demanded by the population. The candidate is then encouraged to express his rare differences of opinion with the party line because he is competing with the other candidates from his party.

Other important consequence, we obtain an equal number of electors per riding to within one (1) voter. Between each election, it is very easy to rebalance the size of the electorate attributed to each riding. Combined to a proportional representation, this system allows each vote to recover an equal weight. Note that the electoral systems with moderate proportional representation [1] or compensatory proportional representation [2] mitigate this injustice too, but to a lesser extent.

The last repercussion of this step is the attribution of candidates to ridings or the determination of opponents. Because the circumscription is no longer a reference, it becomes possible to choose other criteria to regroup the opponents of a same riding. Allowing each party to confront its "specialist" against that of the party in power, each riding should generate specific debates. For example, the critic of the official opposition in matters of health, doctor and nurse candidates from third parties could all angle for the riding of the current Minister of Health. We have seen before that several of them could be elected (component No. 3-B).

In chronological order, here is how we could produce the list of opponents. At first, the party in power publishes its list of 100 candidates. The party in power should be ready to defend against anyone the politics it had the opportunity to apply. In decreasing order of votes at the last election, each party opposes a maximum of 100 candidates to the candidates already present. The official opposition thus opposes an opponent to each
candidate from the party in power. Then the third party proposes a candidate in each riding, knowing which pair it will have to face, and so on... The independent candidates can run after all the candidates of official parties. Finally, the riding numbers are attributed randomly by the general election management.

## Impact on behaviors and implementation

The electoral system resulting from these five components is proportional and without circumscription. Therefore, the results should better represent the will of the electors by reducing the random effects and the biased strategies. In addition, this system is totally applicable to a complementary election.

During the electoral campaign, debates occur on a global basis and ideas from all over the country/province/municipality confront one another. The elector only has to follow the relevant debates in his riding as reported by the media. He can align his vote according to the positions and subjects discussed by the leaders or the candidates of his riding. He can also change his choices using the preferential ballot. And by encouraging the candidates to express their rare differences of opinion in relation to the party line in front of equivalent population samples, we do not measure the preferences between circumscriptions anymore, but rather the opinion and priorities of the population between different debates. More debates (free-trade, pro-life/pro-choice, sovereignty, ...) become possible and different opinions are thus voiced: the elector can express himself. An elector can also signify that he rejects all proposed candidates. Hence, an election is equivalent to numerous small referendums whereas in the current system, the debate often focuses on a single subject.

Besides, this electoral system is more representative, more fair some would say. It is unfair that another candidate comes to destroy this representation by dividing the vote of one of the previous options. Even so, it is also fair that this other option be able to get represented. The preferential ballot combined with the rallying formula allows for all these philosophies to be opposed, then regrouped. Hence, a multitude of candidates is desirable without being harmful. It is the population who decides which ideas do not deserve any representation. More specifically, the proportional representation of the suggested model allows independent candidates to remain and the "crutch" ensures a viable government without tripartite (or more) coalition.

This model also gets rid of a recurrent task: the apportionment into electoral divisions, which often constitutes a dissension between the parties as well as an administrative burden for the Direction Générale des Élections (DGE) who always tries to keep circumscriptions with similar populations. A system without geographical ridings would prevent any discrete injustice (gerrymandering) where for example a party looses most of its seats $49 \%$ against $51 \%$ and wins some $95 \%$ against $5 \%$ with an average support of $60 \%$. It would not prevent the appointment of ministers to treat regional problems. In addition, by attributing ridings at random, we can avoid having the same people always choosing or refusing a future Prime Minister.

Finally, each candidate would have the opportunity to get elected by the quality of his ideas and his discourse, and not by his ability to favour his electorate or by the arbitrariness of electoral fate (the circumscription where he runs, the presence of independent candidates or other parties dividing the vote, etc.). Electoral bargaining should vanish by itself. Therefore, this electoral system should improve behaviors in politics. All electors' votes count, directly through a proportional representation and at each voting round to designate the last candidate. There are no more Z type districts (won in advance or without hope) where the elector cannot express himself.

Of course, such a system can be implemented gradually, beginning with some attemps over small homogeneous municipalities, less reluctant to the idea of ridings without geographical representation. Compared to FPTP, this model allows the electors to indicate their order of preference between the candidates, it avoids any injustice due to the size or the delimitation of circumscriptions and it banishes electoral bargaining. Compared to other proposed methods, the proportional representation of this voting system is optimal and no arbitrary parameter is introduced. For example, the compensatory proportional model of the German electoral system [2] does not prevent vote splitting. When the proposed model is compared with the proportional model with list [3], it is the electorate who builds the list according to its own criteria by voting for candidates: thus, we avoid the unstoppable election of those at the top of the lists.

It is possible to use only some of the features of the currently described system. The "crutch" can very easily be added to the current FPTP electoral system to avoid shaky multiparty coalitions and winner reversal. As a second step, the preferential ballot, the rallying formula and the proportional sytem previously described can be added in order to obtain an optimal proportional representation. These two steps already constitute a clearly superior electoral system that will hopefully be considered by the Commission on electoral reform. Finally, the last step consists in suppressing the notion of geographical division, thus removing any form of injustice about the weight of every vote. Of course, this is the component that the political class is the less willing to accept. Nevertheless, the combination of these components is the one that produces the desired characteristics. All you need is to see past the strange and unusual aspect of this voting system to benefit from its qualities...

Any comments would be appreciated.

## References:

[1] Vincent Lemieux, Policy Options Politiques, vol.18, No.9, Nov/nov. 1997: p.12-15.
[2] Henry Milner, Making Every Vote Count; Reassessing Canada's Electoral System, ouvrage collectif, Peterborough, Ontario Broadview Press, 1999, 200 pages.
[3] Paul Cliche, Pour réduire le déficit démocratique: le scrutin proportionnel, Montréal, Les Éditions du renouveau québécois, 1999, 153 pages.
[4] Louis Massicotte, Pour une réforme du système électoral Canadien, Choix, IRPP, vol.7, No.1(f), Février 2001.
[5] Mouvement pour une Démocratie Nouvelle, http://www.democratie-nouvelle.qc.ca.
I wish to personally thank Mr. André Blais, professor in political sciences at Université de Montréal, and Mr. François Soumis, professor in applied mathematics at École Polytechnique, for their useful advice. I would also like to thank all other contributors for their involvement.
(C) copyrights of Stéphane Rouillon, 2002.

