The art of voting scientifically

by Nick Doe

Voting systems often produce results that are arguably unfair. The problems include:

- mismatches between the number of seats political parties win and their popularity among the general electorate
- vote-splitting by like-minded candidates
- inability of minor parties to win seats unless their support is very localized.

These problems arise in Canada because of our inheritance of British voting practices. Few other democracies use these, and even in the UK, only the House of Commons remains with the old system.[1] Alternative voting systems are used to elect members to the Scottish Parliament, the National Assembly for Wales, the Northern Ireland Assembly, and the European Parliament. Reform to a more proportional electoral system is under active consideration for the parliament in London.[2]

In this article, I want to consider three different voting situations:

1. one contest for a single position
2. one contest for several positions, both with and without party political involvement
3. lots of simultaneous contests, with party politics playing a major role—a parliamentary election.

I am not going to describe every possible voting system—there are far too many for that to be possible—but I will (I hope) cover most of their general principles.

Single-position elections

Let's first consider the process used to elect one person to a particular office. Examples are president of the golf club, Mayor of Gabriola (if that's not the same thing), and an MP or MLA in a single-member riding.

If the result of an election were:

A	*B*
60 votes	40 votes

it's a safe bet that most people (who aren't Irish) would feel that *Candidate A*, with 60 of the 100 votes, should be given the job.

But let's say the election result is:

A	*B*	*C*
40 votes	36 votes	24 votes

Maybe everyone would be happy if *Candidate A* got the job, maybe not. There are two situations at least where the other candidates deserve consideration.

The first situation is fairly common. It arises when *Candidates B* and *C* have split the "anti-A" vote. Voters who voted for *Candidate C* might, in the event that *C* loses, much prefer that *Candidate B* got the job, and vice versa. These voters, with a combined total of 36 + 24 = 60 votes, would be upset with a result that declared Candidate *A*, with only 40 votes, the winner.

The second situation arises less often, but is worth considering. Suppose *Candidate A* and *Candidate B* stand for diametrically opposed political positions—supporters of these candidates would be bitterly disappointed, even very angry, if the other

[1] Smith-Hughes, *Guide to voting systems in the UK*, http://www.charter88.org.uk/pubs/brief/vote_guide.html

[2] *The Report of the Independent Commission on the Voting System*, (known as "The Jenkins Report"), HMSO, October 29, 1998.

candidate were to be elected. Maybe in these circumstances, there exists a consensus for electing a compromise candidate, *Candidate C*, even though this candidate is the first choice of only 24% of the voters.

Many people have studied different voting systems, and although there is no general agreement as to what is the best system—they all have their flaws—there is general agreement that the "first passed the post" (FPTP) system is close to being one of the worst systems for determining what it is that voters want. When the candidate with the most votes (a *plurality*) does not have the support of the *majority*, we need more information. We need to know what voters would like to happen in the event that the candidate they most favour can't get enough votes for an outright win.

Run-off voting

The alternative system of voting for a single office that is most familiar to Canadians is the *run-off* voting system (RVS). It's the system political parties use to elect their leaders. If no candidate wins a majority, the candidate with the fewest votes is eliminated and another round of voting takes place. This process continues until there is a majority winner.

Having several rounds of voting is not practical when it comes to public voting because it is expensive, and the experience of countries like France, which uses a two-round RVS in presidential elections, is that voter turnout often falls sharply in the second round. Many people it seems, are just not so fascinated by the process that they want to turn out more than once for an election, particularly if their second-round vote is the same as their first-round vote.

Various variations of the run-off system, known as *instant run-off* voting systems (IRVS) try to achieve the same result as run-off voting in a single round. These systems are popular among democracies around the world, and it is only tradition and geographic isolation that keeps us going with the old FPTP system.

In IRVS, voters are invited to express not only their first choice of candidate, but also their second, third, and so on choices.[3] The first step in tallying the results is to allocate everyone's first choice to the appropriate candidate. If no candidate has a majority, then the candidate with the fewest first-choice votes is eliminated, and those ballots that have the eliminated candidate as first choice are re-allocated on the basis of those voters' second choice, if any. If there is still no majority winner, another candidate is eliminated and the votes of the eliminated candidate are re-allocated as in the first round of tallying, and so on. Eventually, one candidate will emerge with a majority of the votes. Every voter has one vote, but the vote is transferable from eliminated candidates to candidates still in the race. Unlike run-off voting however, voters have no opportunity to re-think their options as the tallying progresses.

In spite of their popularity, run-off systems suffer from the disadvantage that the election result depends heavily on the order in which candidates are eliminated, and this order may be susceptible to manipulation.[4] Some recent TV game shows that call for contestants to vote each other out of the game demonstrate this very well. Voting strategies become more important than policies. Some critics also object to the fact that the second, third, and so on choices of supporters of eliminated candidates are taken into consideration; yet, only the first

[3] Some systems restrict the number of rounds. The two-round *alternative* vote system is an example.

[4] See Appendix C for a simple example.

choice of those supporting successful candidates is counted. Run-off systems also do not solve the reasonable-compromise problem. *Candidate C* in the "reasonable compromise" scenario will simply be eliminated in the first round of any run-off.

Borda voting

The system for electing single candidates that is most favoured by many academics who have studied voting systems is the so-called *Borda* voting system (BVS).[5] In essence, voters are asked to award each candidate with a number of points, depending on how keen they are to see that candidate elected. For technical reasons I won't go into here, the "best" points system is where the most-favoured candidate is awarded the same number of points as there are candidates, the next most-favoured is awarded one less than this, and so on, the least favoured being awarded only one point. Any unranked candidates aren't given any points. It sounds complicated, which is perhaps its biggest drawback, but it works quite well.

Let's look at simple examples.

In the vote-splitting scenario, suppose that the 40 voters whose first choice is *Candidate A* split 50:50 on their second choice. Suppose also that all of the voters whose first choice is *Candidate B* prefer *Candidate C* to *Candidate A* and, similarly, all of the voters whose first choice is *Candidate C* prefer *Candidate B* to *Candidate A*.

	1st	2nd	3rd
20 votes	*A*	*B*	*C*
20 votes	*A*	*C*	*B*
36 votes	*B*	*C*	*A*
24 votes	*C*	*B*	*A*

[5] After the 18th-century French mathematician, Jean-Charles de Borda.

Candidate A wins in the FPTP system with 40 first-choice votes, but under the BVS, the voting choices are:

	A	*B*	*C*
1st	40 votes	36 votes	24 votes
2nd	0 votes	44 votes	56 votes
3rd	60 votes	20 votes	20 votes

Hence the scoring (3 points for first, 2 points for second, and 1 point for third) is:

	A	*B*	*C*
1st	120	108	72
2nd	0	88	112
3rd	60	20	20
Total	180	216	204

and *Candidate B* wins with 216 points.

If you look carefully at the results, you will see that the majority of voters (36 + 24 = 60%) prefer *Candidate B* to *Candidate A*. The majority of voters (20 + 36 = 56%) also prefer *Candidate B* to *Candidate C*. This justifies *Candidate B*'s win.

In the reasonable compromise scenario, suppose that *Candidate C* is the second choice of all voters whose first choice is not *Candidate C*. Suppose also that the 24 voters whose first choice is *Candidate C* split 50:50 on their second choice.

	1st	2nd	3rd
40 votes	*A*	*C*	*B*
36 votes	*B*	*C*	*A*
12 votes	*C*	*B*	*A*
12 votes	*C*	*A*	*B*

Again, *Candidate A* wins in the FPTP system with 40 first-choice votes, but under the BVS, the voting choices are:

	A	*B*	*C*
1st	40 votes	36 votes	24 votes
2nd	12 votes	12 votes	76 votes
3rd	48 votes	52 votes	0 votes

Hence the scoring (3 points for first, 2 points for second, and 1 point for third) is:

	A	*B*	*C*
1st	120	108	72
2nd	24	24	152
3rd	48	52	0
Total	192	184	224

and *Candidate C* wins with 224 points.

Again, if you do the sums, you will see that the majority of voters (36 + 12 +12 =60%) prefer *Candidate C* to *Candidate A*, and the majority of voters (40 + 12 + 12 =64%) prefer *Candidate C* to *Candidate B*. This justifies *Candidate C*'s win.

Unfortunately, it doesn't always happen that the BVS winner also beats every other candidate when compared on a one-on-one basis (called *Condorcet* comparisons), but there is also good reason not to worry about the fairness of the BVS in these rare circumstances.[6]

Like all voting systems, without exception, the BVS has a flaw;[7] however, this is not nearly so serious (I think) as some mathematically inclined analysts have suggested. The perceived flaw is that the outcome of the election can be manipulated by the presence, or absence, of minor candidates who have no chance of winning. As shown in Appendix D however, this flaw only manifests itself if all voters register a ranking preference for every single candidate on the election ballot, something that is unlikely to happen in practice.

In a BVS, if voters voting for a candidate that they think is likely to win, only vote for that candidate, then, if they are right, that candidate will win. If voters voting for a candidate that they are not sure can win, vote for an alternative, they will do little harm to their favoured candidates chances. Some "odd" results that critics come up with can also easily be fixed by applying the rule that a candidate who wins a majority of first-choice votes automatically wins regardless of the BVS scoring.[8]

Approval voting

One very simple alternative to the FPTP voting system is known as *approval* voting.[9] In this system, voters are allowed to vote for as many candidates as they like. Approval voting is the way that members of the UN elect the Secretary General. Approval voting has several advantages over FPTP, while retaining the advantage of being simple and easy to understand.

Perhaps the greatest advantage of approval voting is that the method discourages "negative" campaigning tactics. It is in each candidate's interest not to offend the supporters of the other candidates, because they too may be a source of "approval" votes. Approval voting also has the potential of solving any vote-splitting problems. Electors are allowed to vote for all candidates they are willing to go along with, even though this may mean the number of votes they cast exceeds the number of candidates to be elected. Approval voting also gives minority candidates their due. Voting for a candidate one approves of, but who has little chance of winning, is perfectly reasonable in the approval system because the vote is not "wasted".

Unfortunately, in highly partisan elections, everyone may obstinately refuse to approve

[6] See Appendix B. Marie Jean Antoine Nicolas de Caritat, Marquis de Condorcet, was an 18th-century French mathematician and a contemporary of Borda

[7] Arrow's theorem, see Appendix A.

[8] In the BC provincial election, 2001, 19 of the 79 elected MLAs failed to win a majority in their constituency.

[9] Not "AV". AV usually means "alternative vote" which is a run-off system restricted to two rounds.

anyone but their favourite candidate. The system then becomes no different from FPTP.

Multiple-position elections

When several candidates are to be elected, to a municipal council or in a multi-member constituency, for example, vote-splitting and the poor showing of compromise candidates is less of a concern because voters have more than one vote. In the FPTP system, the number of votes each elector has equals the number of candidates to be elected.

Approval voting, which can be used no matter how many candidates are to be elected, is one alternative possibility, but like FPTP, approval voting falls apart as soon as political parties become involved. If there are, say, six seats at stake, and both *Party A* and *B* field six candidates, and supporters of the two parties only vote for their six candidates, then it is perfectly possible for one of the parties to win all six seats with only just fractionally more than 50% of the voters' approval. When this starts to happen, it's time to move on to some sort of *single transferable* voting system (STVS).[10]

Single transferable voting systems

There are many STV systems—we discussed the simplest one (IRVS) when there is only one position to be filled above. When several candidates are to be elected, as in, for example, a municipal election, the general principle of STV systems is that all candidates must reach a *quota* of votes to be elected. When candidates receive more than the necessary quota, they are allowed to transfer their "surplus" votes, according to voters' wishes, to other candidates. Conversely, candidates who are eliminated are allowed to transfer their "wasted" votes to other candidates.

There is much discussion among voting theorists on the exact details of how quotas are calculated, and how the re-allocation of votes is to be performed, but in practice these details seldom make any difference to the outcome. Usually the quota is the number of votes cast divided by the number of seats, and the re-allocation is done by first distributing all the "surplus" votes as candidates are elected one by one, and then distributing the "wasted" votes as candidates are eliminated one by one.

For example, to illustrate one method of distributing surplus votes, suppose the quota is 42 votes and the partial result of the election is:

A>*B*>*C*	40 votes
A>*C*>*B*	20 votes

This means that the first choice of 40 voters is *Candidate A*, their second choice is *Candidate B*, and their third choice is *Candidate C*. Twenty other voters rank the candidates *A*, then *C*, then *B*. Suppose also that no other voters vote for *Candidate A*.

Under the STVS, *Candidate A* would be deemed elected with 42 votes (the quota) with 40 + 20 – 42 = 18 surplus votes. These surplus votes would be counted as 12 extra votes for *B*>*C*, and 6 extra votes for *C*>*B* because the voters whose first choice is *Candidate A* have shown a 2:1 preference for *Candidate B* over *Candidate C* for their second choice. It's then up to the other voters, the ones who did not vote for *Candidate A*, to determine whether either of *Candidates B* and *C* has achieved the quota.

To illustrate the distribution of wasted votes, suppose the partial result is:

[10] You could use the Borda voting system to elect several candidates, but nobody does; it's too complicated I guess.

A>B>D 20 votes
B>D>A 10 votes

Suppose also that no other voters rank *Candidate D*. Then, because *Candidate D* has no first-choice votes, *Candidate D* is deemed eliminated. In this case, these particular 30 votes would subsequently be counted as 20 extra votes for *A>B* and 10 extra votes for *B>A*, with no preference expressed for other candidates.

As you can see, the math can get quite involved, but that doesn't matter because the principle is sound and uncomplicated, and we have computers to help with the details. In a single-contest election, if 60% of the electorate supports a party that contests all the vacancies, then under the STV system, close to 60% of their candidates should have little problem in getting elected.

Parliamentary elections

If the "first passed the post" (FPTP) system of electing individual candidates is bad, then a system that goes on to use the same principle in a geographically-based ward or constituency multi-contest system is even worse. Suppose for example, that there are five political parties, receiving equally about 20% of voter support. Then, in theory, it is possible for one party to win every single seat—100% representation—just by having slightly more than 20% of the support in every contest.

Practical examples of the unfairness of the system abound. In the 1997 election in the UK, one party received over 810 000 votes, but failed to elect a single MP; but another party, with just 161 000 votes, elected four MPs. In the recent BC election, the 42% of the voters who did not vote Liberal managed to elect less than 3% of the MLAs. Compare this with 1996 when the NDP won with only 39% of the popular vote.

On the federal scene, vote splitting between the Canadian Alliance and Progressive Conservatives is becoming a perennial theme; and not long ago the separatist Parti Quebecois, because of the geographical concentration of their votes in Quebec, managed to form the country's official opposition. These anomalies mostly result from the deficiencies of the voting system: yet, instead of reform, politicians dissipate no end of energy trying to figure out how to win within a flawed system.

When it comes to considering alternative voting systems for parliamentary elections, it has to be recognized that many people, probably most, value the strong link with their local MP or MLA. It is therefore not practical to consider an alternative system that does not preserve this element of the voting system. The solution is to have a *hybrid* system where everyone has two votes. One vote is cast for a local representative (as at present), and the second vote is to be cast either for a political party (the *closed* system), or for a candidate who is not tied to any particular riding (the *open* system). Some of those elected will be local representatives, and some will be a representative of the whole country, province, city, or whatever.[11] That way, supporters of the smaller parties can join together to vote for candidates they like, without the artificial restriction that unless they live in the same place, they have no chance of getting anyone elected.

Notice also, that although in provincial and national elections, most people vote for a party, not a candidate, the hybrid system allows voters to support a strong local candidate, regardless of political affiliation.

[11] Some systems elect regional rather than "global" representatives with the second vote, but the idea is the same.

and, with their second vote, support their favoured political party.

The details that must be worked out before implementing a hybrid or *additional member* voting system (AMVS) include the following:[12]

What proportion of the members should be additional members? In Scotland it is 43%, in Wales it is 33%. The Green Party of BC is proposing 50%.[13] The choice is somewhat arbitrary—the higher the percentage the more closely will the percentage of seats won by a party match the percentage of people who voted for that party. Obviously, minor parties favour a higher percentage than major parties, because major parties do very well, thank you, within the present FPTP system. Notice also that if a proportional system is used to elect individual MPs and MLAs, BVS for example, then the percentage that are additional members need not be so high.

Should the list of additional members be closed (determined by political parties) or open (the choice of the voter)? In both the closed and open systems, the additional candidates are elected from party lists starting with the candidate at the top of the list. The only difference is that in the closed system, the order candidates appear on the lists is announced, in advance of the election, by each party; while in the open system the order candidates appear on the lists is determined by the voters.[14]

The usual procedure (there are others) for allocating the additional member votes to the parties is to determine which party would have the fewest seats per additional vote cast for that party if the additional seat were to be given to that party. For example, if the result of an election were:

Party	Additional Votes	Seats	(Seats +1) per vote
A	60	8	0.150
B	40	2	0.075
C	20	0	0.050

Party C, with a low score of 0.050 seats per vote, would get the first additional seat.

In the next distribution, the situation would now be:

Party	Additional Votes	Seats	(Seats +1) per vote
A	60	8	0.150
B	40	2	0.075
C	20	1	0.100

and *Party B* would get an extra seat.

Gradually, as additional seats are awarded, the number of seats per vote becomes the same for all parties.

In the example above, ten seats are awarded on a single-member-riding basis. If there were an additional, say, five "top-up" seats, the final result would be:

Party	Additional Votes	Seats	(Seats +1) per vote
A	60	8	0.150
B	40	5	0.150
C	20	2	0.150

Party A with 50% of the support would go from 8 seats out of 10 (80%) to 8 seats out of 15 (53%);

Party B with 33% of the support would go from 2 seats out of 10 (20%) to 5 seats out of 15 (33%); and

Party C with 17% of the support would go from 0 seats out of 10 (0%) to 2 seats out of 15 (13%).

[12] *Hybrid* systems are sometimes called *mixed member proportional* voting systems (MMPVS).

[13] *Free your vote* [BC Green Party Initiative], http://www.freeyourvote.bc.ca

[14] Some systems give electors the choice of selecting candidates or leaving it to the party to do that.

It's not perfect, but that's only because constituency members out-number additional members 2:1. *Party A* would, nevertheless, have its wings clipped as its majority of six dwindled to one, as is arguably all it deserves considering it only got 50% of the popular vote. By winning two additional members, *Party C*, which was initially unrepresented in spite of winning 17% of the popular support, would at least get a voice in the proceedings.

Ensuring diversity

One problem with parliamentary voting systems that gets relatively little attention is that of defining a riding. If the parliament uses a "one member–one vote" principle, there ought to be roughly the same number of voters in each riding. The problem is of course that urban population densities are high, and, in comparison, rural population densities are low. While it may be numerically correct to leave rural electors with few representatives, most people consider it unfair if they literally have no say in matters that are to them very important.[15]

The traditional answers have been either to have increasingly large rural constituencies, or to gloss over the inequality and accept that rural members are elected by fewer voters than their city counterparts. A similar problem is currently facing the European Parliament as more countries join the European Union, some much smaller, and with smaller economies, than the founding members.

A possible solution to this type of problem is to abandon the goal of equal-population constituencies, and instead have a weighted voting system. The greater the population a member of parliament represents, the greater the weight of the member's vote in parliament. Without a computer, counting weighted votes would be a nightmare for parliamentary clerks—but then, these days, computers are not difficult to come by.

But is it fair?

In order to analyse "scientifically" whether an election result is "fair", you have to have a mathematical definition of what constitutes fairness. Suppose for example that the result of a parliamentary election is:

Party A	*Party B*	*Party C*
48 seats	47 seats	5 seats

Is it "fair" that *Party C*, representing 5% of the voters, holds the balance of power? Italians have no problem answering that one—their system has been notoriously unstable for years because only weak minority governments have managed to win. Other European countries however use exactly the same system without difficulty.

To answer the question, is it fair? it would be fair if the seat distribution accurately reflected the support for the various parties, and if those elected carefully considered and voted for proposals on their merits, without regard for the "party politics" of the situation. Obviously that's not going to happen; sooner rather than later, the parties are going to start "doing deals" in the manner of international ice-dancing competition judges. Supporters of the FPTP system argue, that while it may result in polarization and dramatic swings from one side of the political spectrum to the other, at least governments are given a clear mandate when it is their turn to take office.

To make a case that the best system may be one that mixes FPTP results with the "pure"

[15] In Canada, one could even extend the problem to cover "virtual constituencies", comprising Aboriginal and non-Aboriginal voters living in the same area.

proportional vote, here are the results of three recent elections. The columns show:

- the actual (FPTP) result
- the result based solely on popular support (100% proportionality)
- the hypothetical result of a hybrid system (2/3 FPTP, 1/3 proportional).

I'll leave it to you to look at the tables and make your own judgement as to what's fair.

Federal election, 2000

	FPTP	Popular support	2/3 1/3
Lib.	172	123	156
CA	66	77	70
PC	12	37	20
BQ	38	32	36
NDP	13	26	17
Other	0	6	2
Lib. majority	43	–55	11

BC Provincial election, 2001

	FPTP	Popular support	2/3 1/3
Lib.	77	46	67
NDP	2	17	7
Green	0	10	3
Unity	0	3	1
Marijuana	0	3	1
Other	0	0	0
Lib. majority	75	13	55

BC Provincial election, 1996

	FPTP	Popular support	2/3 1/3
NDP	39	30	36
Lib.	33	31	32
Reform	2	7	4
PDA	1	4	2
Green	0	2	1
Other	0	1	0
NDP majority	3		–3
Lib.majority		–13	

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Appendix A : Arrow's theorem

Mathematical economist Kenneth Arrow proved in 1952 that it is not possible to invent a "fair" voting method for choosing a winner from three or more candidates. Arrow's four measures of fairness were:

1. a candidate with the majority of first-choice votes should win
2. the winner should be preferred to all other candidates when compared on a one-on-one basis (Appendix B)
3. the winner should remain the winner if voting preferences are changed in favour of the winner (Appendix C)
4. the winner should remain the winner if the votes for one or more of the losing candidates are discarded (Appendix D).

Arrow proved that all voting systems, even those not yet invented, will inevitably fail to pass all four tests.

Appendix B : Condorcet comparisons

Condorcet's proposal for a voting system is that the winner is the one who beats every other candidate when candidates are compared on a one-on-one basis.

For example, if the results are:

	1st	2nd	3rd	4th
45 votes	*A*	*B*	*C*	*D*
30 votes	*B*	*C*	*D*	*A*
25 votes	*C*	*B*	*A*	*D*

Then, looking at votes for *Candidate B*, there are:

55 votes $B>A$, 45 votes $A>B$; *B* beats *A*

75 votes $B>C$, 25 votes $C>B$; *B* beats *C*

100 votes $B>D$, 0 votes $D>B$; *B* beats *D*

Candidate B is therefore the Condorcet winner having beaten all other candidates in pairwise comparisons. *Candidate A* would have been the FPTP winner.

One of the flaws of Condorcet comparisons is that they can produce no result, or an anomalous result. Suppose for example, the election result is:

	1st	2nd	3rd
10 votes	*A*	*B*	*C*
10 votes	*B*	*C*	*A*
10 votes	*C*	*A*	*B*

20 votes $A>B$, 10 votes $B>A$; *A* beats *B*

20 votes $B>C$, 10 votes $C>B$; *B* beats *C*

20 votes $C>A$, 10 votes $A>C$; *C* beats *A*

Now, there is no reason to regard the election results as illogical; yet, if you look at the Condorcet comparisons, you see we have *A* beating *B*; *B* beating *C*; from which one could reasonably deduce that *A* should beat *C*. Yet, that is not what happens, *C* beats *A*.

Although individual votes make sense, when combined for Condorcet comparisons, they may collectively make no sense. This is because the information about how individual groups voted has been lost. The comparisons consequently can produce "strange" results, or, quite often, no result at all.[16]

Appendix C : Run-off voting anomalies

As a simple example of a run-off voting system anomaly, consider the following results:

	1st	2nd
12 votes	*A*	*C*
10 votes	*B*	
9 votes	*C*	*B*

In the first round *C* is eliminated, and *B* easily goes on to beat *A* in the second round, 19:12.

Now suppose the election is run again and five first-round votes are transferred from *A* to *B* with no second-round preference. This seems quite harmless; *B* after all was the winner. Yet the result is now:

	1st	2nd
7 votes	*A*	*C*
15 votes	*B*	
9 votes	*C*	*B*

In the first round, *A* drops out, and so *C* wins the second round, 16:15.

All that happened was votes were transferred to the winner, but as a consequence, the winner lost.

Appendix D : Borda voting's weakness

Here is an example of the Borda voting system's sensitivity to the participation in the election of a so-called "irrelevant" candidate:

[16] Donald G. Saari,
The symmetry and complexity of elections,
http://www.colorado.edu/education/DMP/voting_b.html

Suppose the voting preferences are:

	1st	2nd	3rd
45 votes	*A*	*C*	*B*
30 votes	*B*	*A*	*C*
25 votes	*C*	*B*	*A*

then the candidates' rankings are:

	A	*B*	*C*
1st	45	30	25
2nd	30	25	45
3rd	25	45	30

and the points scored (3 for first place, 2 for second, and 1 for third) are:

	A	*B*	*C*
1st	135	90	75
2nd	60	50	90
3rd	25	45	30
Total	220	185	195

Candidate A wins with 220 points.

Now introduce *Candidate D* who is thought by everyone to be a weak candidate, but who is a favourite of the supporters of *Candidate B*. These voters rank *B* and *D* as a pair in that order, so that when we re-run the election we get:

	1st	2nd	3rd	4th
45 votes	*A*	*C*	*B*	*D*
30 votes	*B*	*D*	*A*	*C*
25 votes	*C*	*B*	*D*	*A*

	A	*B*	*C*	*D*
1st	45	30	25	0
2nd	0	25	45	30
3rd	30	45	0	25
4th	25	0	30	45

Under BVS rules, the points scored are (4 for first place, 3 for second, 2 for third, and 1 for fourth):

	A	*B*	*C*	*D*
1st	180	120	100	0
2nd	0	75	135	90
3rd	60	90	0	50
4th	25	0	30	45
Total	265	285	265	185

Candidate B now wins with 285 points, all thanks to *Candidate D*'s participation.

While acknowledging that in theory such anomalous results are possible, we have also to ask how likely are these scenarios? Analysts who have spent some time simulating elections and monitoring the results, say "not very". Look again at the last example and suppose that 60% vote for *Candidate D* in the way suggested, but 40% think *Candidate D* is hopeless and leave him or her unranked (shown darker in the table). Then the voting goes like this:

	1st	2nd	3rd	4th
27 votes	*A*	*C*	*B*	*D*
18 votes	*B*	*D*	*A*	*C*
15 votes	*C*	*B*	*D*	*A*
18 votes	*A*	*C*	*B*	
12 votes	*B*	*A*	*C*	
10 votes	*C*	*B*	*A*	

	A	*B*	*C*	*D*
1st	45	30	25	0
2nd	12	25	45	18
3rd	28	45	12	15
4th	15	0	18	27

Under BVS rules, the points scored are (4 for first place, 3 for second, 2 for third, and 1 for fourth):

	A	*B*	*C*	*D*
1st	180	120	100	0
2nd	36	75	135	54
3rd	56	90	24	30
4th	15	0	18	27
Total	287	285	277	111

Candidate A remains the winner.

Real voting situations are obviously likely to be complicated and simple assumptions about the outcome invalid. Unfortunately, not many jurisdictions have ever tried the BVS, so there is little hard evidence about what does happen in practice.

In conclusion, let me add that alternative voting systems are currently a hot topic. You'll find lots more information on the web. ◊